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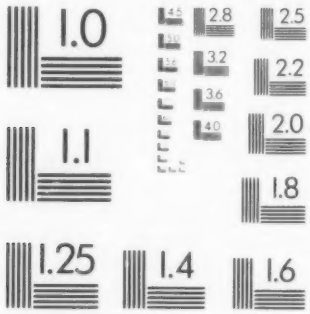

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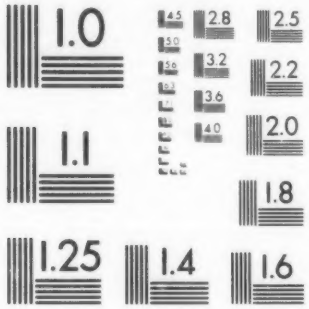

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Working papers are circulated in the language of preparation, to make analytic work undertaken by the staff available to a wider readership. They have received only limited evaluation and do not reflect the views of the Department of Finance. Comments on the working papers are invited and may be sent to the author(s).

Does the Length of the Current Expansion Increase the Probability of Recession?

by
Lisa Backa

Working Paper No. 96-06

Economic and Fiscal Policy Branch
Department of Finance, Canada

The author wishes to thank Jeremy Rudin, Cliff Halliwell and Sheila MacDonald for their valuable comments and suggestions. Any remaining errors and omissions are the sole responsibility of the author.

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Abstract

This paper examines whether there is any reason to believe that the expansion phase of the Canadian business cycle is more likely to end as time passes. Three issues are analysed -- whether there is evidence of "duration dependence" in business cycle phases, whether the pattern of business cycle durations has changed over time, and whether the length of one phase helps to predict the length of the next. Based on preliminary empirical work for Canada and a review of research completed for the U.S., three results, common to the post-war business cycles of both countries, emerge. First, contrary to popular opinion, the probability of entering a recession does not increase significantly as the expansion ages. Second, the average duration of expansions has increased over time. Third, long contractions tend to be followed by long expansions. All three results support this conclusion: the fact that the current expansion is already relatively long does not imply that a recession must be around the corner.

Résumé

Dans ce document de travail, on se demande s'il y a des raisons de croire que la phase d'expansion du cycle économique canadien est vouée à prendre fin avec le temps. Trois questions sont analysées : des faits prouvent-ils la «dépendance à l'égard de la durée» dans les phases du cycle économique; la tendance a-t-elle changé au fil du temps en ce qui concerne la durée des phases du cycle économique; la durée d'une phase permet-elle de prévoir la durée de la phase suivante. Trois constatations, propres aux cycles économiques de l'après-guerre des deux pays, ressortent des travaux expérimentaux préliminaires sur le Canada et d'un examen des travaux de recherche menés sur les États-Unis. Premièrement, contrairement à la croyance populaire, la probabilité d'une récession n'augmente pas sensiblement du fait que la phase d'expansion se prolonge. Deuxièmement, la durée moyenne des phases d'expansion a augmenté. Troisièmement, les périodes de contraction prolongées ont tendance à être suivies de périodes d'expansion prolongées. Les trois constatations amènent à conclure que la phase d'expansion en cours déjà relativement longue ne signifie pas qu'une récession pointe à l'horizon.

I. Introduction

With the Canadian economy currently in its fifth year of expansion and the U.S. economy in its sixth, many comments have been made in the press that these 'long' expansions are increasing the probability of recession. For example, in the spring of 1996, a private forecasting firm stated that the odds clearly favoured a recession in Canada sometime in the next five years, because if there is *not* a recession in the next five years, the current non-recessionary period would equal the longest non-recessionary period since the Second World War. Is there any credibility to the notion that the expansion phase of the business cycle is more likely to end as time passes?

To answer this question, this paper analyses three issues. The first is the idea of duration dependence. This specifically asks whether the length of time the economy is in one state of the business cycle (either expansion or contraction) influences the probability that the state will be terminated. While references to average expansion length and average contraction length are frequently used alongside references to the probability of switching from one state to another, empirical tests can determine whether or not expansions or contractions typically 'die of old age'.

The second issue examined is whether the pattern of business cycle durations has changed over time. In the U.S., it is generally argued that expansions have become longer and contractions shorter since the Second World War, but the duration of whole cycles has not changed. A similar shift in Canadian durations would provide further evidence that comparing current durations to earlier ones is misleading.

The third stream of research examined is whether the length of one phase of a business cycle is related to the length of the next phase. In the U.S. and Canada, long expansions tend to be followed by short contractions. If there was a positive relationship between the length of expansions and the length of preceding contractions, this could provide further evidence that the length of the current expansion is not out of the ordinary, given the length of the last recession.

While a great deal of empirical research has been completed on these topics using U.S. data, no empirical research has been published for Canada. However, the results for the U.S. are relevant to Canada to the extent that the Canadian economy is very closely tied to the U.S., with periods of expansion and contraction in the two tending to overlap. In addition, the U.S. research provides a framework for completing empirical work for Canada. As a result, this paper reviews the empirical approaches and conclusions for the U.S. and then summarises some preliminary empirical work for Canada.

The outline of this paper is as follows. Section I examines duration dependence, Section II considers whether the pattern of business cycles has changed over time, and Section III examines whether the length of one phase of a business cycle affects the length of the next. Section IV summarises the findings and the implications for Canada.

II. Duration Dependence

To introduce the issue of duration dependence, consider a simple coin toss with two outcomes, heads or tails. An observer might believe that a run of consecutive heads or consecutive tails would be more likely to end as the number of consecutive tosses increased. However, while the run may seem more likely to end, the underlying probabilities are constant over time: the probability of a head remains constant at 0.5 and the probability of a tail remains steady at 0.5. In addition, while the average length of runs of consecutive heads and of consecutive tails over time can be calculated, the averages have no predictive power about how long an existing run will continue.

This type of analysis can be extended to business cycles. Does the length of time the economy is in one state of the business cycle (either expansion or contraction) influence the probability that the state will be terminated?

If expansions or contractions are more likely to end as they get older, they are said to exhibit positive duration dependence. For example, if the probability of a recession increases with the length of an expansion, then expansions are said to exhibit positive duration dependence. However, if the probability of a recession does not change with the duration of an expansion, then expansions exhibit no duration dependence.[1]

II.1. Empirical Evidence for the U.S.

Summary

Economists have pursued many different streams of empirical research to study the issue of duration dependence, including different types of parametric and non-parametric estimation. A stylized fact emerges from the studies of duration dependence in U.S. business cycles: expansions before World War II (i.e. prewar) and contractions after World War II (i.e. postwar) tend to exhibit statistically significant positive duration dependence. Thus, *in the postwar period, the length of a recovery does not significantly increase the probability of a recession*, but the probability of an expansion increases as a recession ages.

The asymmetry between prewar and postwar duration dependence in expansions and contractions may perhaps be attributed to a change in macroeconomic policy following the War. The start of the postwar period saw a significant strengthening of powers of, and public commitment to use, monetary and fiscal policies to stabilize the economy. If policymakers perceived a link between the durations of expansions and contractions and welfare, discretionary government policy in the postwar period may have attempted to reduce contractions and increase expansions. In particular, if political pressure on

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[1] The absence of duration dependence is also termed a 'Monte Carlo' business cycle. Just as some casino gamblers incorrectly *perceive* a run of good or bad luck as having predictive power on future bets, some business cycle observers *perceive* that the duration of an expansion or contraction provides information on the probability of switching states.

policymakers increased as a contraction aged, then contractions could exhibit positive duration dependence.

Policymakers may also have attempted to lengthen expansions in the postwar period, with expansions ending due to either policy mistakes or a tightening to reduce inflation. If policy mistakes and tightening were as likely for short expansions as for long expansions, then post-war expansions would not exhibit duration dependence.

Various Estimation Approaches

A. Non-parametric Estimation

Hazard functions, frequently used to analyse unemployment durations, provide a convenient tool for estimating duration dependence in business cycles. The hazard function, $\lambda(\tau)$, is defined as the *conditional* probability that a state ends at duration τ, given that it has achieved duration τ. A constant hazard function (such as $\lambda_1(\tau) = \lambda$ in Chart 1) indicates that the probability of exiting the state remains constant over all durations, and is thus independent of the length of time in the state. Thus, a constant hazard corresponds to the notion of no duration dependence. In contrast, an increasing hazard (such as $\lambda_2(\tau) = \gamma\tau$ in Chart 1) indicates that the probability of exiting the state increases with time, consistent with positive duration dependence.[2]

Since the unconditional density of durations, $f(\tau)$, can be defined in terms of the hazard function, $\lambda(\tau)$,

$$f(\tau) = \lambda(\tau)\exp\left[-\int_0^{\tau}\lambda(u)du\right],$$

any given hazard function implies a particular density.[3] For example, a constant hazard implies an exponential density of durations, $f(\tau) = \lambda \exp(-\lambda\tau)$ (Chart 2). Thus, when the termination probability is constant over time, the density is monotonically declining.

A non-constant hazard implies a non-exponential density of durations, with the specific density depending on the functional form of the hazard. For example, the increasing hazard, $\lambda_2(\tau)$ above, implies a density $f(\tau) = \gamma\tau \exp(-(\gamma/2)\tau^2)$ (Chart 2). In general, the probability mass associated with an increasing hazard is more concentrated around its mean than that associated with the exponential distribution of the same mean.

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[2] A decreasing hazard means that the probability of exiting the state decreases with the duration of the state, consistent with negative duration dependence.

[3] For further details, see Kiefer (1988).

Chart 1:
Constant and Increasing Hazard Functions

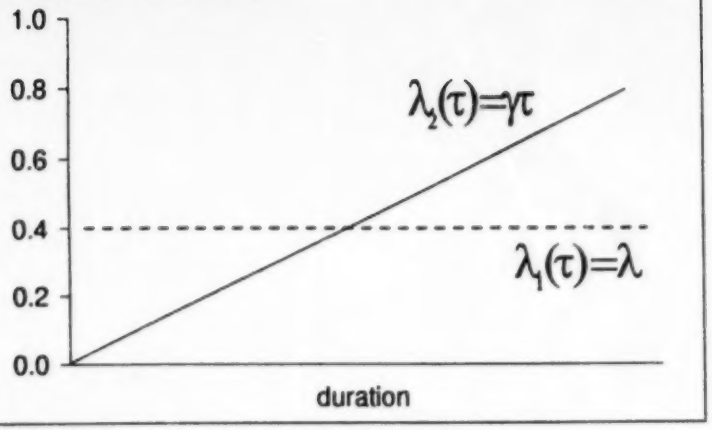


Chart 2: Duration Densities

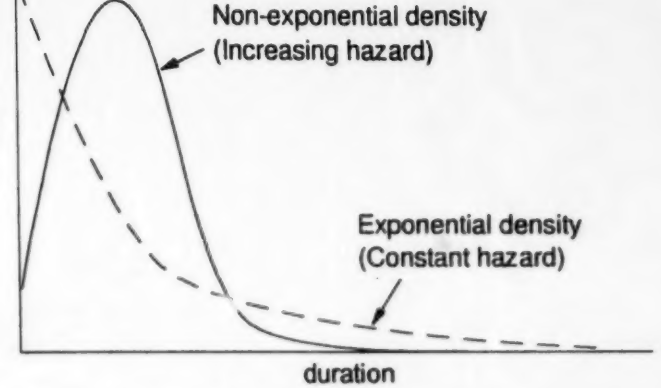
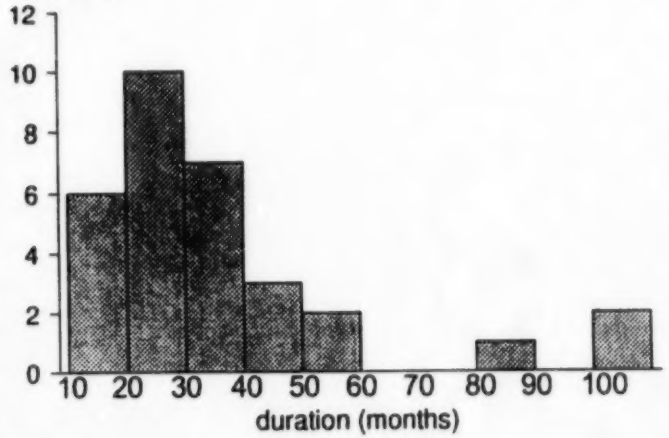


Chart 3: Histogram of U.S. Expansion Durations



Non-parametric estimation approaches test observed durations of expansions and contractions for conformity to the exponential distribution. For example, do the durations of U.S. business cycle expansions (Chart 3)[4] conform to the exponential distribution?

With non-parametric approaches, the functional form of the hazard is not specified. This may be an advantage, as incorrect parameterizations of the functional form of the hazard can lead to misleading inferences. However, non-parametric approaches only indicate whether or not there is evidence of duration dependence, without providing an estimate of its magnitude.

Early non-parametric estimation of duration dependence was based on chi-squared goodness of fit tests. These approaches compared estimated sample histograms to a null of the exponential distribution (reflecting a constant hazard). These included work by McCulloch (1975), Anderson (1977), Savin (1977), deLeeuw (1987) and Diebold and Rudebusch (1987). Only deLeeuw found evidence of duration dependence.

A few explanations have been put forth to explain the lack of support for duration dependence in this earlier research. First, these non-parametric tests may have lower power in small samples. Second, the test results are sensitive to the (arbitrary) histogram construction. The shape of the histogram is affected by both the interval length (e.g. 5 months or 10 months) and the interval definition (e.g. from 0 to 10 to 20 months, or from 5 to 15 to 25 months).

Diebold and Rudebusch (1990) completed further estimation using different non-parametric techniques with higher power than their earlier goodness of fit tests, and did find some evidence of duration dependence.[5] They directly tested the observed durations for conformity to the null of the exponential distribution. They used Shapiro and Wilk's (1972) W-statistic test for the exponential null[6] and a modified W-statistic test for the

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[4] Chart 3 is based on data from 1854 to 1991 (Annex Table 1). The height of each bar in the chart represents the number of durations that were recorded within the range specified on the x-axis. For example, 6 expansions lasted between 10 and 19 months, 10 expansions lasted between 20 and 29 months, etc.

[5] While Diebold and Rudebusch noted that, based on Monte Carlo studies, their non-parametric tests had good power in small samples, they still recognised that small sample size may impair the power of their tests.

[6] Renumbering the observed durations, x, of each sample in ascending order, the W test statistic is constructed as:

$$W = \frac{(\bar{x} - x_1)^2}{(N-1)\sigma^2} \text{ , where } \bar{x} = \sum_{i=1}^{N} \frac{x_i}{N} \text{ and } \sigma^2 = \sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N}$$

exponential distribution conditional on a minimum duration, γ.[7] The latter test was considered because the NBER, in defining business cycle dates, only designates cycles that have reached a minimum maturity duration which does not strictly accord with the exponential distribution. Diebold and Rudebusch also considered Brain and Shapiro's (1983) test statistic for the exponential distribution, Z, and an alternate statistic, Z^*, which was more sensitive to alternative duration distributions associated with non-linear hazard functions.

In each of the non-parametric tests, the null hypothesis is no duration dependence (consistent with the exponential density or truncated-exponential density of durations). Thus, evidence of duration dependence is indicated by small p-values which reject the null and indicate significant departures from exponentiality. The p-values corresponding to the various test statistics for various samples are shown in the table below. Prewar expansions exhibited significant positive duration dependence (with p-values all less than .05) but postwar expansions did not exhibit significant duration dependence.[8] Neither prewar nor postwar contractions exhibited any evidence of duration dependence at conventional significant levels (p-values were all well above .10), but there was more evidence of duration dependence (positive) in postwar contractions.

Table 1: P-values for Diebold and Rudebusch's Non-parametric Tests

Sample	**Statistic**			
	W	**W(t_0=γ)**, with γ=10 for expansions γ=6 for contractions[9]	**Z**	**Z***
Expansions:				
Prewar	.015	.044	.033	.008
Postwar	.420	.633	.547	.320
Contractions:				
Prewar	.904	.859	.974	.067
Postwar	.188	.580	.268	.351

---

[7] Defining $A = \sum_{i=1}^{N}(x_i - \gamma)$ and $B = \sum_{i=1}^{N}(x_i - \gamma)^2$, the modified W statistic is

$$W(t_0 = \gamma) = \frac{A^2}{N\left[(N+1)B - A^2\right]}$$

[8] The direction of the duration dependence is indicated by the sign of the Z statistic, with positive duration dependence associated with a negative Z statistic. Note that it is the p-values which are displayed in Table 1.

[9] Diebold and Rudebusch set the minimum duration equal to the shortest observed durations of expansion (10 months) and contractions (6 months) and checked the robustness of their results using other values for the minimum duration.

B. Parametric Estimation

Unlike non-parametric approaches, parametric estimation of duration dependence not only tests for duration dependence but, if it exists, also provides an estimate of its magnitude. In addition, parametric techniques may have higher power than non-parametric techniques in detecting duration dependence. However, the disadvantage of parametric approaches is that a specific functional form (e.g. for the hazard) must be assumed. If an incorrect parametric form is imposed, the results may be misleading.

Many different parametric approaches have been followed in the estimation of duration dependence. These include hazard functions, regime-switching models, and modelling turning points.

B.1. Hazard functions

In addition to being used in non-parametric estimation, hazard functions can be used to test for duration dependence using parametric techniques. Parametric tests specify a functional form of the hazard function, including a parameterization of duration dependence. The functional form chosen must be able to nest the constant hazard case corresponding to no duration dependence. The parameters of the hazard can be estimated using the method of maximum likelihood, and one can test whether the duration dependence is statistically significant or not. Of particular value is that the associated empirical hazards can then be calculated using the parameter estimates to determine whether the duration dependence is also economically significant.

Sichel (1991) was the first researcher to apply parametric hazard models to the analysis of business cycle duration dependence. The functional form of the hazard which Sichel chose was:

$$\lambda(\tau_i) = \mu\alpha\tau_i^{\alpha-1}$$

where τ_i represents the duration of the ith observation, μ is a constant, and α parameterizes the duration dependence. Note that if α is greater than 1, then the hazard (the conditional probability of a turning point occurring) increases as the phase ages, and there is positive duration dependence. If α equals 1, then the hazard collapses to a constant hazard ($\lambda(\tau_i)=\mu$).

This hazard parameterization implies a Weibull duration density, which collapses to the exponential distribution if α equals 1. Two additional factors -- censoring and truncation -- must be considered in the construction of the likelihood function. Some observed durations may be censored, in that they are not completed. For example, the current business cycle expansion marks a censored observation because the exact length of the spell is not observed. The only information a censored observation provides is that the spell is at least a certain number of periods long, t_i. Thus, the contribution of a censored observation to the likelihood function is the survivor function -- the probability

that the duration is longer than t_k. The contribution of each uncensored observation (completed spell) to the likelihood function is the density function.

In addition, durations may be truncated if there is a minimum maturity criteria that must be met before an expansion or contraction is identified. Since the NBER imposes a minimum maturity criteria when classifying U.S. business cycles, an adjustment for truncation must be incorporated in the likelihood function. Using the hazard function specified above and accounting for censoring and truncation yields the following likelihood:

$$\ln L = \sum_{i=1}^{N} \left\{ \log(\alpha\mu) + (\alpha - 1)\log(d_i) - \mu(d_i^{\alpha} - d_0^{\alpha}) \right\} - \mu(d_N^{\alpha} - d_0^{\alpha})$$

where the d_i represent observed completed durations, d_0 represents the minimum maturity criteria, and the d_N represent the duration to date of censored observations.

To test for duration dependence, the parameters of the hazard are estimated using maximum likelihood and then the estimate of α is tested to see whether it is significantly greater than 1. The parameter estimates in Table 2 indicate that positive duration dependence was evident in all prewar, postwar and full samples of expansions and contractions. However, only the prewar expansions and postwar contractions exhibited *statistically significant* positive duration dependence.

Table 2: Sichel's Hazard Model Parameter Estimates
(standard errors in brackets)

	Expansions			**Contractions**		
	Full	Prewar	Postwar	Full	Prewar	Postwar
α	1.277 (0.293)	2.379* (0.518)	1.278 (0.603)	1.230 (0.279)	1.345 (0.354)	3.020* (1.070)
μ	0.0117 (0.014)	0.0003 (0.001)	0.0067 (0.018)	0.0336 (0.034)	0.018 (0.024)	0.0007 (0.002)

*significantly different from unity at 5 per cent level

To assess the *economic significance* of duration dependence, the corresponding empirical hazard functions were calculated using the above parameter estimates and various durations. From the results, one can judge whether the rise in the probability of exiting the state is deemed to be significant. The results (Table 3) indicate that the samples that exhibited statistically significant duration dependence also exhibited economically significant duration dependence. For example, the following illustrates the stark contrast in the results of prewar and postwar expansions: The probability that a *prewar* expansion will end in the next month, given its duration to date, rises from 1 per cent after 9 months

to 39 per cent after 96 months, while the probability that a *postwar* expansion will end is fairly constant, only rising from 2 per cent to 3 per cent over the same number of months. A large difference also arises in the results for prewar and postwar contractions, with the empirical hazard for prewar contractions relatively flat, rising only from 5 per cent to 7 per cent over 9 months. In contrast, for postwar contractions, the hazard rises from 18 per cent to 73 per cent over the same number of months.

Table 3: Sichel's Empirical Hazard Functions

Duration (months)	9	12	18	24	36	48	60	72	96
Expansions									
Prewar	.01	.02	.04	.06	.10	.15	.20	.26	.39
Postwar	.02	.02	.02	.02	.02	.03	.03	.03	.03
Contractions									
Prewar	.05	.06	.07						
Postwar	.18	.32	.73						

Recognizing that the hazard model estimated by Sichel (1991) imposed restrictions on the admissible hazard function shapes, Diebold, Rudebusch and Sichel (1993) developed an exponential-quadratic hazard model, which can be viewed as a low-ordered series approximation to an arbitrary hazard:

$$\lambda(\tau_i) = \exp(\beta_0 + \beta_1 \tau_i + \beta_2 \tau_i^2) \quad ,$$

where τ_i represents the duration of the *i*th observation.

This hazard model allowed three different hazards to be estimated for each sample: the exponential-constant case (β_1=β_2 =0) corresponding to no duration dependence, the exponential-linear case (β_2=0) and the exponential-quadratic case. Each of these three hazards was interpreted, in turn, as an increasingly flexible approximation to the true hazard.

The corresponding likelihood function is:

$$\ln L = \sum_{i=1}^{I} \left[d_i (\beta_0 + \beta_1 \tau_i + \beta_2 \tau_i^2) - \int_0^{\tau_i} \exp(\beta_0 + \beta_1 x + \beta_2 x^2) dx \right]$$

where d_i equals 1 if the *i*th observation is uncensored and 0 if censored.

The maximum likelihood parameter estimates for each of the three hazard models are displayed in Table 4. The models were compared using Wald tests. The p-values for the null that the particular coefficient equals zero are shown in brackets. The small p-values

for the null that $\beta_1=0$ in the exponential linear models for prewar expansions (.001) and postwar contractions (.027) indicate statistically significant departures from the constant hazard model in each case. Therefore, prewar expansions and postwar contractions were found to exhibit statistically significant duration dependence.

However, the large (above .10) p-values associated with the null that $\beta_1=0$ in the other two samples indicate that the constant hazard null cannot be rejected for prewar contractions and postwar expansions at conventional significance levels. Further, the quadratic term does not play an important role in any sub-sample, as the estimates are small. In addition, the p-values associated with the null of $\beta_2=0$ are all well above .10.

Table 4: Diebold, Rudebusch and Sichel's Hazard Model Parameter Estimates*

	β_0	β_1	β_2
Expansions: Prewar			
Constant	-3.0		
Linear	-3.9	.06 (.001)	
Quadratic	-4.4	.13	-.0017 (.181)
Expansions: Postwar			
Constant	-3.9		
Linear	-4.2	.010 (.223)	
Quadratic	-4.3	.018	-.0001 (.433)
Contractions: Prewar			
Constant	-2.8		
Linear	-3.0	.014 (.172)	
Quadratic	-3.3	.056	-.0009 (.201)
Contractions: Postwar			
Constant	-1.7		
Linear	-2.7	.20 (.027)	
Quadratic	-2.7	.24	-.0034 (.460)

*the p-values associated with the Wald statistics in the exponential linear and exponential quadratic models are in brackets

To consider the economic implications, the empirical hazards corresponding to these parameter estimates were calculated at various durations (Table 5).[10]

Table 5: Diebold, Rudebusch and Sichel's Empirical Hazard Functions

Duration (months)	12	18	24	36	48	72	96
Expansions: Prewar							
Linear	.027	.039	.055	.113	.231		
Quadratic	.022	.041	.067	.124	.139		
Expansions: Postwar							
Linear	.016	.017	.018	.020	.022	.028	.035
Quadratic	.015	.016	.018	.021	.024	.029	.032
Contractions: Prewar							
Linear	.055	.060	.065	.077	.091	.127	
Quadratic	.053	.067	.080	.093	.085	.033	
Contractions: Postwar							
Linear	.278	.897					
Quadratic	.287						

These results are comparable to Sichel's (1991) results. The hazards for prewar expansions rise fairly sharply, while the hazard for postwar expansions, regardless of whether the linear or quadratic models is used, remains fairly flat, rising only from .02 after 12 months to .03 or .04 after 96 months. The hazard for prewar contractions rises slowly while the hazard for postwar contractions jumps from .3 to .9 over 6 months using the linear model.

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10 The constant hazard for prewar expansions was .051, for postwar expansions was .021, for prewar contractions was .062 and for postwar contractions was .177.

B.2. Regime-switching models

The notion that business cycles consist of two phases, expansions and contraction, lends itself to econometric analysis based on regime-switching models. Such models allow the time series dynamics to be governed by a parameter vector that switches, depending on the regime (or state) that is realized. In this context, real GDP growth can be modelled as switching between two states (expansions and contractions) with each recording a different mean growth rate.

Hamilton's (1989) regime-switching model for post-war U.S. real GNP growth is the starting point for many extensions dealing with duration dependence. Hamilton assumed a stochastic process for output y_t with a potentially-switching mean (with a state (S) of 0 representing a contraction and a state of 1 representing an expansion):

$$y_t = \mu(S_t) + \sum_{i=1}^{4} \phi_i (y_{t-i} - \mu(S_{t-i})) + \sigma v_t ,$$

with $v_t \sim N(0,1)$, $\mu(S_t)=\alpha_o$ if $S_t=0$ and $\mu(S_t)=\alpha_o+\alpha_1$ if $S_t=1$.

Regime-switching models also include a description of the probability law governing the change from one state to another. Hamilton assumed that state transitions were governed by a first-order Markov process, so that the probability of moving from one state to another (the state transition probability) depended only on the most recent state, as follows:

$$P_{ij} = P(S_t = j | S_{t-1} = i) = P(S_t = j | S_{t-1} = i, S_{t-2} = k, \ldots, S_0 = m) ,$$

with probability P_{ij} representing the probability that state i will be followed by state j. For example, P_{oo} represents the probability of staying in the contractionary state, and P_{11} represents the probability of staying in the expansionary state.

Thus, Hamilton assumed that transition probabilities were constant (fixed) over the sample. These constant transition probabilities implied that the expected durations of expansions and contractions were also constant over the sample, calculated as $(1/(1-P_{11}))$ and $(1/(1-P_{00}))$, respectively.

Maximum likelihood estimation indicated that an expansion would be followed by another quarter of expansion with a probability of 0.9 and the probability of continuing a contraction was 0.76. Thus, expansions exhibited more persistence than contractions. These estimated transition probabilities implied that the expected complete duration of an

expansion was constant at 10 quarters while the expected complete duration of a contraction was constant at 4 quarters.[11]

Durland and McCurdy (1994) extended Hamilton's model by defining state transitions as a higher-order Markov process. This allowed state transition probabilities to depend not only on the most recent (inferred) state, but also on the number of periods, d, the system has been in the state, as follows:

$$P(S_t = j|S_{t-1} = i) = P(S_t = j|S_{t-1} = i, D_{t-1} = d)$$

These conditional state transition probabilities were parameterized using a logistic functional form,[12] as follows:

$$P_{ij} = P(S_t = j|S_{t-1} = i, D_{t-1} = d) = \frac{\exp(a(i) + b(i)d)}{1 + \exp(a(i) + b(i)d)} \quad for\ i = j = 0,1$$

$$= 1 - \frac{\exp(a(i) + b(i)d)}{1 + \exp(a(i) + b(i)d} \quad for\ i \neq j$$

Explicitly introducing this duration variable, d, into the model allowed the statistical significance of duration dependence to be tested. The maximum likelihood parameter estimates relevant for our analysis are displayed in Table 6.

Table 6. Durland and McCurdy's Markov Model with Duration Dependent Transition Probabilities

Parameter	Estimate	Robust Standard Error	t-statistic
a(0)	6.516	2.055	3.17
a(1)	4.305	2.363	1.82
b(0)	-1.348	.296	-4.55
b(1)	-.243	.282	-.86

The parameter estimates of duration dependence in both recessions (b(0)) and expansions (b(1)) are negative. Based on the state transition probabilities specified above, this implies that both expansions and contractions are more likely to end as they progress. As

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[11] The parameter estimates and the observed data were used to infer the unobserved states (recession or expansion). The inferred states corresponded closely to the NBER business cycle dates.

[12] This functional form was chosen because it ensured that all probabilities would lie between 0 and 1, it ensured that the sum of the probabilities over all states for each t would equal 1, and it allowed the nesting of a first-order Markov model with constant transition probabilities.

the estimate of b(0) is larger in absolute value than that of b(1), recessions exhibit stronger duration dependence than recessions. Further, only the estimate of b(0) is statistically significant; the estimate of b(1) is not. Thus, for post-war U.S. data, only recessions exhibit statistically significant positive duration dependence.

For each possible duration of expansion and contraction (d), the parameter estimates were used to calculate the conditional probability of continuing in the same state. Durland and McCurdy's results indicated that the conditional probability of continuing an expansion remained fairly constant above 0.9, while the conditional probability of continuing the contraction dropped off sharply as the duration increased (from 0.97 after 2 quarters to close to zero after 9 quarters).

Durland and McCurdy (1994) tested their Markov model (which allowed duration dependence in the state transition probabilities) against Hamilton's model (which held state transition probabilities constant). Using a likelihood ratio test, Hamilton's model was rejected for their model with duration dependence.

While Durland and McCurdy allowed the transition probabilities to vary with the duration to date of the state, there are other ways of incorporating time-varying transition probabilities (TVTP). For example, Filardo (1994) and Filardo and Gordon (1993) extended Hamilton's Markov-switching model to allow the state transition probabilities to vary with economic-indicator variables from the goods and financial markets, including the Commerce Department's Composite Index of Leading Indicators (CLI). The state transition probabilities were defined as:

$$P(S_t = j | S_{t-1} = i) = P(S_t = j | S_{t-1} = i, z_t),$$

where z_t is a set of economic-indicator variables. These transition probabilities were parameterized as:

$$P_{11}(z_t) = \frac{\exp(\theta_{p0} + \sum_{j=1}^{J1} \theta_{pj} z_{t-j})}{1 + \exp(\theta_{p0} + \sum_{j=1}^{J1} \theta_{pj} z_{t-j})}$$

$$P_{00}(z_t) = \frac{\exp(\theta_{q0} + \sum_{j=1}^{J2} \theta_{qj} z_{t-j})}{1 + \exp(\theta_{q0} + \sum_{j=1}^{J2} \theta_{qj} z_{t-j})}$$

with the probability of exiting the state equal to one minus the probability of continuing in the state.

Maximum likelihood estimation indicated that the economic-indicator variables had a statistically significant impact on the transition probabilities. The model with time-

varying transition probabilities fit the data better and characterized business cycle fluctuations better than a fixed transition probability model. Again, expansions had higher persistence than recessions.

Filardo and Filardo and Gordon found that post-war contractions exhibited positive duration dependence. They also found that the expected duration of contractions tends to fall dramatically near the end of a recessionary period and the expected duration of expansions also falls towards the ends of expansions. They highlighted the fact that time-varying transition probability models allowed the expected duration of a phase to vary across time, which is preferable to forcing the expected durations to be constant as the fixed transition probability models (such as Hamilton's) did.

Lahiri and Wang (1994), in another extension of Hamilton's Markov-switching model, evaluated the CLI as a predictor of business cycle turning points, using both real-time and historically revised data. Like Hamilton, they assumed that transition probabilities were independent of phase duration. From their maximum likelihood estimates of the population parameters, including the transitional probabilities, they inferred the unobserved state which they then used to forecast turning points. While they did not explicitly test for duration dependence, they generated the transition probabilities over the sample period by estimating the Markov model recursively by adding one monthly observation at a time as they were available in real time. Lahiri and Wang found that the transition probabilities did not systematically vary over the sample which suggested to them that the probabilities were not influenced by the duration to date of the state. From this, they concluded that there was no evidence of duration dependence.

Kim and Yoo (1995) estimated a multivariate Markov-switching factor model of coincident economic indicators. While they did not explicitly introduce a duration variable into the model, they were able to test for duration dependence indirectly. To do so, they compared the estimation results of a constant transition probability model with the results of a time-varying transition probability model (where the transition probability depended on economic indicators). They found evidence of positive duration dependence in postwar U.S. contractions.

B.3. Modelling Turning Points

Johnson (1987) also found evidence of duration dependence in postwar U.S. contractions in quarterly business cycle chronology. Using a parametric approach, he found that phase duration to date had additional explanatory power in predicting turning points after controlling for fiscal and monetary shocks.

II.2. Other International Empirical Evidence

Empirical research based on the estimation of hazard functions has been extended internationally to France, Germany, and Great Britain. Similarities in the duration dependence characteristics of business cycles over countries have been discovered. In

particular, the stylized fact evident in the U.S. data has also been found in the European pre-war data.[13]

Diebold, Rudebusch and Sichel (1990) extended the parametric and non-parametric approaches used in Diebold and Rudebusch (1990) and Sichel (1989, 1991) to test for duration dependence in pre-war data for France, Germany and Great Britain. Pre-war results for France, Germany, and Great Britain were similar to those in the U.S. All four countries exhibited statistically and economically significant positive duration dependence in expansions but not in contractions. Diebold, Rudebusch and Sichel concluded that the similarity in the pre-war pattern of duration dependence across these four countries suggested conformity in the characteristics of business cycles.

Further international results (consistent with the earlier findings) were presented in Diebold, Rudebusch and Sichel (1993). Applying the exponential-quadratic model (outlined above) to pre-war data for France, Germany and Great Britain led to the same results as those for the U.S. All expansions exhibited strong positive duration dependence, with departures of the hazards from constancy highly significant. In contrast, all estimated contraction hazards were nearly constant, with deviations from constancy not significant.

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[13] Research for Europe has been limited to pre-war data because reconstruction after the second world war led to difficulty in measuring business cycles in the same manner as before the war. In particular, there has been a scarcity of true recessions in the postwar period.

II.3. Empirical Evidence for Canada

No empirical research has been published for Canada. However, there are many factors which suggest the business cycle in Canada may exhibit patterns of duration dependence similar to those found in the U.S. The Canadian business cycle is very closely tied to the U.S. economy. Notice that periods of expansion and contraction in the two economies are very similar and tend to overlap (Charts 4 and 5).[14] In addition, the similarities in the duration dependence characteristics in the U.S. and the European countries suggests that Canadian business cycles may also exhibit similar patterns of duration dependence.

A. Non-parametric Estimation

Some non-parametric tests were completed for Canada following the approach of Diebold and Rudebusch (1990). Although one drawback of non-parametric tests is that they may have lower power than parametric tests, the particular tests employed by Diebold and Rudebusch had more power than other non-parametric tests.

Recall that Diebold and Rudebusch tested observed durations of expansions and contractions for conformity to the null of the exponential distribution, using W-statistic and modified W-statistic tests. These statistics were calculated for the durations of Canadian expansions and contractions, based on the Statistics Canada Chronology from 1947 to 1992 (Annex Table 2).

The p-values corresponding to the W- and modified W-test statistics for Canadian expansions and contractions were all well above 0.1 (lying between 0.5 and 0.9, Table 7). These p-values indicate that the null hypothesis of no duration dependence could not be rejected at conventional significance levels. Therefore, these tests found no evidence of duration dependence in Canada in either expansions or contractions.

Table 7: P-values for Non-parametric Tests for Canada

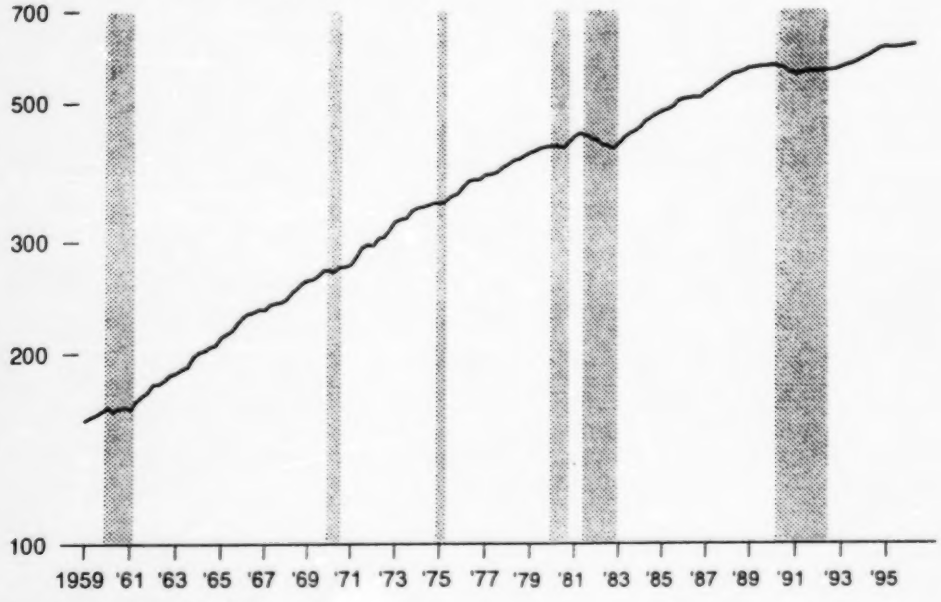
Sample	**Statistic**	
	W	**$W(t_0=\gamma)$** , with γ=10 for expansions γ=3 for contractions[15]
Expansions	between 0.5 and 0.9	between 0.5 and 0.9
Contractions	between 0.5 and 0.9	between 0.5 and 0.9

[14] The NBER Business Cycle Chronology for the U.S. is found in Annex Table 1. The Statistics Canada Business Cycle Chronology for Canada is found in Annex Table 2.

[15] Following Diebold and Rudebusch, the minimum duration was set equal to the shortest observed durations of expansion (10 months) and contractions (3 months).

Chart 4: Canadian Real GDP*

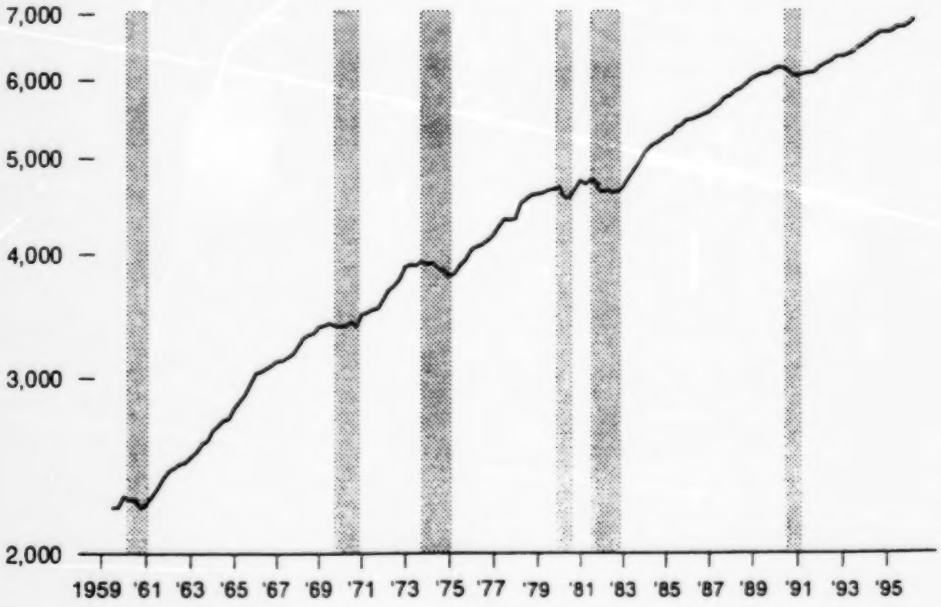
billions of 1986 dollars



*contractions are shaded

Chart 5: U.S. Real GDP*

billions of 1992 dollars



*contractions are shaded

B. Parametric Estimation

Some parametric estimation, using hazard functions, was also completed for Canadian business cycles. Following Sichel (1991), the hazard function was specified as:

$$\lambda(\tau_i) = \mu\alpha\tau_i^{\alpha-1}$$

where τ_i represents the duration of the ith observation, μ is a constant, and α parameterizes the duration dependence. Recall that if α is greater than 1, then the hazard increases as the phase ages, and there is positive duration dependence. If α equals 1, then the hazard collapses to a constant hazard.

In the construction of the likelihood function, an adjustment must be made for censored Canadian durations. However, an adjustment for truncation need not be made because, unlike in the U.S., Statistics Canada does not hesitate to identify short business cycle durations. For example, a contraction of 3 months is recorded in 1975.

Using the hazard function specified above and accounting for censoring yields the following likelihood:

$$\ln L = \sum_{i=1}^{N} \left\{ d_i \log(\alpha\mu) + (\alpha - 1) d_i \log t_i - \mu(t_i)^{\alpha} \right\}$$

where d_i equals 1 if the observation is uncensored and 0 if it is censored and t_i represents the duration of the ith observation.

To test for duration dependence, the parameters of the hazard are estimated using maximum likelihood and then the estimate of α is tested to see whether it is significantly greater than 1.

Positive duration dependence is evident in both expansions and contractions (as the estimates of α are positive, Table 8). To test the sensitivity of the results to the inclusion of the censored observation (the current expansion), the second set of estimation results for expansions reflect only the completed durations. The expansion results are similar regardless of the sample used. While there is evidence of positive duration dependence in all samples, neither the expansions nor the contractions exhibit *statistically significant* positive duration dependence at conventional significance levels. Unfortunately, the Canadian business cycle chronology only extends back to 1947, so a prewar and postwar comparison is not possible.

Table 8: Hazard Model Parameter Estimates for Canada
(standard errors in brackets)

	Expansions (incl. censored)	**Expansions (uncensored only)**	**Contractions**
α	1.4205 (.60058)	1.3879 (.58330)	1.7211 (.55762)
μ	.0037784 (.0090217)	.0047851 (.011298)	.015402 (.024087)

To assess the *economic significance* of duration dependence, the corresponding empirical hazard functions are calculated using the above parameter estimates and various durations. From the results (Table 9), one can judge whether the rise in the probability of exiting the state is deemed to be substantial.

Table 9: Empirical Hazard Functions for Canada

Duration (months)	**3**	**9**	**12**	**18**	**24**	**36**	**48**	**60**	**72**	**96**	**108**
Expansions											
Incl. censored	.01	.01	.02	.02	.02	.02	.03	.03	.03	.04	.04
Uncensored	.01	.02	.02	.02	.02	.03	.03	.03	.03	.04	.04
Contractions	.06	.13	.16	.21	.26	.35	.43	.51	.58	.71	.78

The duration dependence in expansions does not appear to be economically significant, as the hazard for expansions is very flat. This is true regardless of whether the censored observation is included or not. The probability that an expansion will end in the next month, given its duration to date, only rises from 1 per cent (or 2 per cent) after 9 months to 4 per cent after 96 months. These results are very similar to the U.S. results, where the postwar expansion termination probability increased from 2 per cent after 9 months to 3 per cent after 96 months.

While the hazard for contractions is much steeper than for expansions, the probability does not increase substantially during reasonable durations. Recall that the longest contraction duration recorded in Canadian history is 25 months, while the average is 10 months. The probability of contraction termination rises from 13 per cent after 9 months to about 26 per cent after 25 months. Further, the hazard for U.S. postwar contractions is much steeper, as it rises to 73 per cent after only 18 months.

To assess the sensitivity of the Canadian results to business cycle dating techniques, the estimations were also completed for the original Statistics Canada business cycle chronology, which only extends from 1950 to 1981 (see Annex Table 3).[16] Hazard function parameter estimates based on the original (six) observations only are displayed in Table 10. For comparison, the subset (seven) of the revised observations corresponding to the same time period, 1950 to 1981, were also re-estimated. In both samples, the qualitative results are the same as those reported earlier, that is, positive duration dependence is evident in both expansions and contractions, but it is not statistically significant.

Table 10: Hazard Model Parameter Estimates for Canada, Original and Revised Chronologies
(standard errors in brackets)

	Expansions		**Contractions**	
	Original	**Revised**	**Original**	**Revised**
α	1.1173 (.72535)	1.5027 (.65414)	4.9373 (2.949)	2.1215 (1.2211)
μ	.011937 (.040022)	.0029209 (.0079795)	.000008 (.00006)	.0092723 (.024568)

The empirical hazard functions corresponding to the original and revised durations from 1950 to 1981 are displayed in Table 11. Similar to the hazard for the entire sample of revised expansion durations (Table 9), the hazard based on the revised durations over the smaller sample also rises to 4 per cent after 96 months. The hazard based on the original expansion durations rises slowly from .015 after 3 months to .023 after 108 months. For contractions, the hazard for the original durations rises quite sharply to 70 per cent after 12 months. While this hazard is much sharper than the revised duration hazard, it is only based on a sample of six observations.

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[16] The original Statistics Canada business cycle chronology was published in Current Economic Analysis, March 1982. The revised chronology was published in the Canadian Economic Observer, February 1996. The revised chronology incorporates both new data and information about business cycles gained from recent experiences. In particular, the original chronology is based on changes in economy-wide domestic output (real GDP) whereas the revised chronology also reflects changes in employment and population.

Table 11: Empirical Hazard Functions for Canada, Original and Revised Chronologies

Duration (months)	3	9	12	18	24	36	48	60	72	96	108
Expansions											
Original	.015	.017	.018	.018	.019	.020	.021	.022	.022	.023	.023
Revised	.01	.01	.02	.02	.02	.03	.03	.03	.04	.04	.05
Contractions											
Original	.003	.23	.70								
Revised	.07	.23	.32	.50	.69						

III. Has the Pattern of Business Cycles Changed?

As mentioned earlier, references to the length of the current expansion relative to the average expansion are used to make inferences about the next turning point. The current Canadian expansion has been underway for 53 months, while the average expansion has lasted only 43 months,[17] leading some observers to suggest that this increases the probability of a recession occurring. However, in order for any comparisons involving average durations to be meaningful, the pattern of business cycle durations must remain fairly unchanged over time. To the extent that the expected duration of expansions has been increasing, comparing recent durations to earlier durations (to predict turning points) may be misleading.

III.1. Empirical Evidence for the U.S.

This stream of research for Canada was motivated by non-parametric and parametric estimation which showed that the pattern of U.S. business cycles has changed since WWII.

A. Non-parametric Estimation

Diebold and Rudebusch (1992) found statistically significant evidence of a postwar shift toward longer expansions and shorter contractions, but no evidence that the duration of whole cycles has changed. They based their results on *non-parametric* Wilcoxon, or rank-sum, tests which rejected (at the 0.1 per cent level) the null hypothesis of no change in the behaviour of durations, for both expansions and contractions. In contrast, for whole cycles they found that the null of no change in duration could not be rejected even at the 20 per cent level. Thus, they concluded that there has been a reallocation of business cycle time from contractions to durations, leaving whole cycles unchanged.

In addition to being statistically significant, Diebold and Rudebusch found the changing pattern of durations to be significant in an economic sense. First, the average duration of expansions had roughly doubled since WWII (from an average duration of 25 months in the prewar period to 50 months in the postwar period) while the average duration of contractions had roughly been halved (from 20 months to 10 months). Second, the amount of time spent in the contractionary state had fallen significantly since WWII. More than 40 per cent of the prewar period was spent in recession while only 20 percent of the postwar period was spent in recession. Third, the ratio of expansion duration relative to the preceding contraction duration increased from an average of 1.5 in the prewar period to 4.5 in the postwar period.

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[17] Based on Statistics Canada's business cycle dates.

Diebold and Rudebusch offered many reasons for the change in the pattern of business cycle durations. These reasons included volatility stabilization around an upward trend,[18] stronger powers of monetary and fiscal policies, as well as an increased public commitment to use monetary and fiscal policies to stabilize the economy. In addition, there were structural changes including an increased share of services (which have a more moderate cycle), increased availability of consumer credit and improvements in inventory management.

B. Parametric Estimation

Sichel (1991) used *parametric* techniques to test whether the average durations of expansions and contraction had changed since World War II. His technique allows for further testing of whether the average durations had *gradually* changed over time. Recall that Sichel's hazard function specification was:

$$\lambda(\tau_i) = \mu\alpha\tau_i^{\alpha-1}$$

where τ_i represents the duration of the ith observation, μ is a constant, and α parameterizes the duration dependence. To test whether the average durations have changed, the location parameter μ is specified as a function of observable variables and then substituted into the likelihood function (specified earlier).

To test for a *discrete* shift in the average duration of expansions and contractions at a given break point, the location parameter is specified as:

$$\mu = \mu_0 \exp(\mu_1 z)$$

where the shift variable z equals 0 before the specified break point and 1 after the break point. If μ_1 equals zero, then there is no change in the average durations after the break point. If the estimate of μ_1 is significantly less than zero, then μ is smaller after the break point. From the hazard function, this implies that the average level of the hazard is smaller after the break point. This in turn implies that the average phase duration is longer after the break point. On the other hand, if the estimate of μ_1 is significantly greater than zero, then the average phase duration is shorter after the break point.

In his test for a discrete shift in the mean phase duration, Sichel took the break point to be the Second World War. The results (displayed in Table 12) indicate that the mean duration of expansions was significantly longer in the postwar period (μ_1 is negative) and

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[18] In the limit with zero volatility around an upward trend, the economy is in permanent expansion growing at the trend rate. Volatility stabilization in the postwar period may reflect many factors, including an increased role of government (including a large and progressive tax system that mitigates cyclical fluctuations in disposable income) and greater availability of consumer credit that reduces liquidity constraints, thus leading to smaller aggregate demand shocks. For an example of such a study, see DeLong and Summers (1986). While the conclusion of postwar volatility stabilization is widely accepted, Romer (1986) argues that the difference in volatility in the prewar and postwar periods largely reflects differences in data construction methods.

the mean duration of contractions was significantly shorter in the postwar period (μ_1 is positive).

Table 12: Sichel's Test for Discrete Shift in Mean Phase Durations

	Expansions	Contractions
μ_0	.001 (.002)	.007 (.008)
μ_1	-1.681* (.490)	1.466* (.470)
α	2.022* (.391)	1.605* (.319)

*Indicates α is significantly different from unity using 5% one-sided test, and any other coefficient is significantly different from zero.

Notice that the estimates of α for both expansions and contractions are now found to be significantly greater than 1, implying positive duration dependence in full samples, contrary to the results presented in Table 2. This difference in results is related to heterogeneity across observations, which biases estimates of α downward. In general, heterogeneity is not viewed as a critical problem in analyzing duration dependence in business cycles, because the prior is positive duration dependence with α greater than 1. Since heterogeneity biases estimates downward, estimates could only have been biased away from positive duration dependence. Since adjustments for a mean shift in durations adjust for one type of heterogeneity across observations, larger estimates of α are expected.

To test for a *trend* shift over time in the average duration of expansions and contractions, the location parameter is specified to include a trend:

$$\mu = \mu_0 \exp(\mu_2 i)$$

where i represents the observation number, i.e. i equals 1 for the first observation, 2 for the second etc. If μ_2 equals zero, then there is no trend in the phase durations. If μ_2 is significantly less than (greater than) zero, then average phase durations are increasingly longer (shorter) over the sample.

Sichel's results (Table 13) indicate that the mean duration of expansions exhibited a statistically significant upward trend over the sample (μ_2 for expansions significantly negative) while the mean duration of contractions exhibited a statistically significant downward trend (μ_2 for contractions significantly positive).

Table 13: Sichel's Test for Trend Shift in Mean Phase Durations

	Expansions	Contractions
μ_0	.005 (.007)	.005 (.006)
μ_2	-.062* (.021)	.061* (.024)
α	1.761* (.357)	1.52* (.308)

*Indicates α is significantly different from unity using 5% one-sided test, and any other coefficient is significantly different from zero.

III.2. Empirical Evidence for Canada

It would be of interest to test the Canadian data for a statistically significant shift in the pattern of business cycle durations over time. Since the Canadian business cycle chronology only extends back to 1947, a prewar and postwar comparison is not possible.[19] However, the Canadian business cycle durations recorded since 1947 could be split into two equal groups, the first sub-sample including the business cycles from 1947 to 1960 and the second sub-sample including the cycles from 1961 to 1990.

A. Non-parametric Estimation

While the Wilcoxon non-parametric test completed by Diebold and Rudebusch for U.S. data could not be used to test the Canadian data,[20] a similar non-parametric test could be applied. As Diebold and Rudebusch state, the null hypothesis that the behaviour of durations has not changed from one sample to the other is equivalent to the null hypothesis that the duration distributions of the two samples are identical. The homogeneity of the Canadian business cycles duration distributions across the two samples (identified above) can be tested with a log-rank test. The results indicate that the distribution of *expansion* durations is significantly different (direction cannot be inferred) in the first and second samples. In contrast, the distribution of *contraction* durations is not significantly different in the two samples.[21]

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[19] The Canadian business cycle chronology begins in 1947 with the start of the quarterly National Accounts data. On a monthly basis, real GDP is not available prior to 1961. However, from 1947 to 1961, monthly industrial production data, together with quarterly real GDP (which provides an economy-wide measure of output), are used to determine cyclical turning points.

[20] The reference papers including the critical values for the test could not be located.

[21] The p-value for expansions is .027. For contractions, the p-value is .43.

Further, the calculations completed by Diebold and Rudebusch to assess the *economic* significance of the change in the U.S. durations can be completed for the Canadian observations. To provide meaningful comparisons with the U.S. data (with only postwar Canadian data available), the calculations based on U.S. durations were redone using the same sample periods as Canada (i.e. 1947 to 1960 and 1961 to 1990).

In both Canada and the U.S., the average duration of expansions has increased sharply from the first sample to the second (Table 14). In Canada, the average duration tripled from 21 months in the first sample to 64 months in the second, while the average duration in the U.S. almost doubled to 63 months in the second sample. In contrast, the average duration of contractions has remained fairly steady from one sample to the next in both countries, rising from 10 months to 11 months. In addition, in both countries, the shift towards longer expansions was accompanied by an increase in whole cycle durations.

Table 14: Average Durations, Canada and the U.S.

	Expansions (months)		**Contractions** (months)		**Whole Cycles** (months)	
	1947-60	1961-90	1947-60	1961-90	1947-60	1961-90
Canada	21	64	10	11	31	75
U.S.	36	63	10	11	46	75

Second, the amount of time spent in the contractionary state has fallen over time, in both countries (Table 15). In fact, the change is more pronounced in Canada, where in the first sample, 32 per cent of business cycle time was spent in recession but this fell to 14 per cent of time in the second sample.

Table 15: Share of Business Cycle Time Spent in State, Canada and the U.S.

	Expansions		**Contractions**	
	1947-60	1961-90	1947-60	1961-90
Canada	68%	86%	32%	14%
U.S.	79%	85%	21%	15%

Third, the ratio of expansion duration relative to the duration of the preceding contraction has increased substantially in Canada, from 2.5 in the first sample to 9.7 in the second. In the U.S., the ratio increased from 3.9 in the first sample to 5.2 in the second.

These preliminary results for Canada do suggest that there has been a move toward increased duration of expansions over time. This implies that comparing recent durations to earlier ones is misleading.

B. Parametric Estimation

Parametric tests for shift in the average duration following Sichel (1991) were also completed for Canada. In the case of the test for a discrete shift in the mean phase duration, whereas Sichel chose the break point to be the Second World War the break point for Canada was chosen to be the midpoint of the recorded durations (so that the sample was split in two).

The discrete-shift results are presented in Table 16. As the estimate of μ_1 for expansions is significantly less than zero, this implies that μ is smaller in the second half of the expansion observations. This in turn implies that the mean duration of expansions was significantly longer in the second half of the observations. In contrast, there is no evidence that the mean duration of contractions has undergone a discrete shift, as μ_1 for contractions is not statistically different from zero.

As with the U.S. estimates, the estimates of α are now higher because adjustments to control for the mean shift in durations adjust for one type of heterogeneity across observations. This lower heterogeneity bias leads to significant positive duration dependence in expansions.

Table 16: Test for Discrete Shift in Mean Phase Durations in Canada

	Expansions[22]	Contractions
μ_0	.00052727 (.0011616)	.014238 (.021414)
μ_1	-2.8249* (.89359)	-.80607 (.85028)
α	2.3927* (.66805)	1.9396 (.55938)

*Indicates α is significantly different from unity using 5% one-sided test, and any other coefficient is significantly different from zero.

The trend-shift results (Table 17) indicate that the mean duration of expansions exhibited a statistically significant upward trend over the sample (μ_2 for expansions is significantly

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22 Including censored observation.

negative). However, the mean duration of contractions has not exhibited a statistically significant trend (μ_2 for contractions not significantly different from zero).

Table 17: Test for Trend Shift in Mean Phase Durations in Canada

	Expansions[23]	Contractions
μ_0	.0090239 (.020861)	.014369 (.02496)
μ_2	-.38769* (.20123)	-.22443 (.14274)
α	1.8125 (.48247)	2.3384* (.72495)

*Indicates α is significantly different from unity using 5% one-sided test, and any other coefficient is significantly different from zero.

[23] Including censored observation.

IV. Effect of the Length of One Phase on the Length of the Next

The most recent recession in Canada (from April 1990 to April 1992) marked the longest recession recorded since business cycles have been dated in Canada (since 1947). If a positive relationship exists between the lengths of expansions and the length of preceding contractions, this would provide evidence that the length of the current expansion is not out of the ordinary, given the length of the previous recession.

IV.1. Empirical Evidence for the U.S.

This stream of research was motivated by the stylized fact evident in the U.S. data: long expansions tend to be followed by short contractions (see Charts 6 and 7). Zellner (1990) suggested that long phases of expansion may reflect strong growth forces at work in the economy. The influence of these growth forces may persist beyond the turning point and act in such a way to allow the adjustments of the contraction phase to take place in a shorter time than if the growth forces had not been present.

Empirical research (Zellner (1990) and Diebold (1993)) has shown there is a statistically significant negative relationship between the length of contractions and the length of preceding expansions in the U.S. data, both prewar and postwar.[24] Diebold's regression results (OLS) are displayed in the table below. Diebold argued that an additional month of expansion tended to be associated with roughly a one-half month reduction in the duration of the following contraction in the prewar period and, in the postwar period, a one-fifth month reduction in the following contraction.

Table 18: Diebold's U.S. Estimation Results, Dependent Variable: Contraction Length

	Prewar	Postwar
Intercept	27.6 (p=.00)	17.5 (p=.01)
Expansion length	-.47 (p=.00)	-.18 (p=.12)
Observations	15	6
$\bar{R}^2$	.57	.38

[24] The estimation is based on the NBER Business Cycle Chronology which is found in Annex Table I. U.S. business cycles are classified into major and minor cycles based on their duration and amplitude. Zellner and Diebold used minor cycles in their estimation because major cycles frequently contain wars which tend to dominate any other features of economic activity.

Chart 6: Durations of Neighbouring Expansions and Contractions in U.S. prewar data

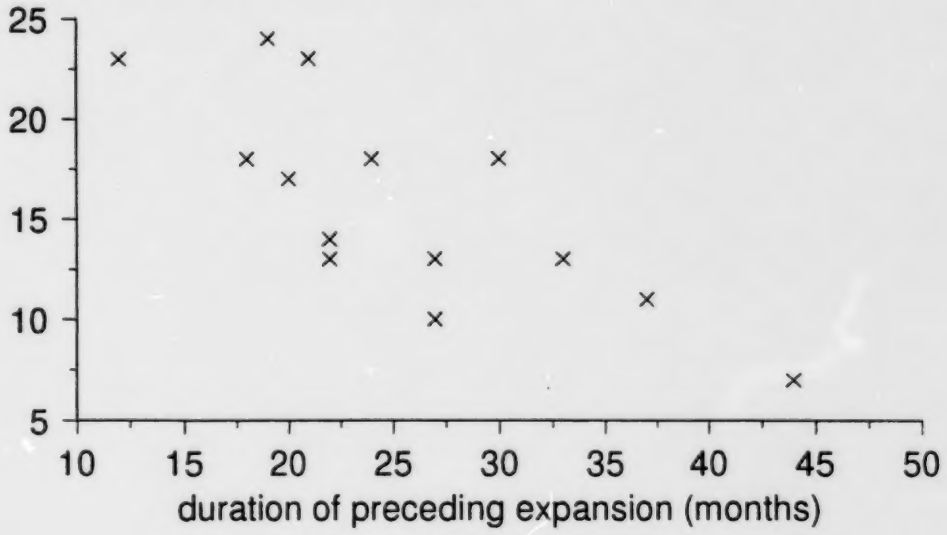
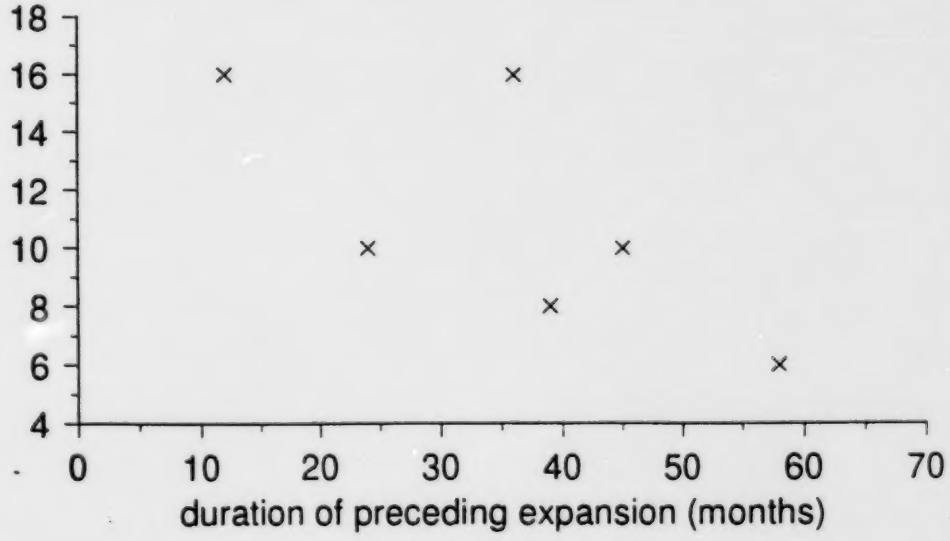


Chart 7: Durations of Neighbouring Expansions and Contractions in U.S. postwar data



Graphical analysis of the U.S. business cycle durations suggests there may also be a relationship between the length of an expansion and the length of the preceding contraction (Chart 8). Therefore, the postwar U.S. data were tested for a relationship in this direction. As the OLS regression results in the table below indicate, the length of a contraction has a positive influence on the length of the following expansion. An additional month of contraction tends to be associated with a 5-month increase in the length of the following expansion (based on the results of the regression excluding an intercept term).

Table 19: U.S. Estimation Results, Dependent Variable: Expansion Length

	Including intercept term	Excluding intercept term
Intercept	-14.0 (p=.74)	
Contraction length	6.2 (p=.13)	5.0 (p=0.0)
Observations	7	7
$\overline{R}^2$	.27	.38

IV.2. Other International Empirical Evidence

Zellner also found a statistically significant negative relationship between the length of contractions and the length of preceding expansions using prewar data for Great Britain. However, he did not find a negative relationship in the prewar data for France or Germany.

IV.3. Empirical Evidence for Canada

Although there are a smaller number of data points than in the U.S., Canadian business cycles appear to exhibit the same stylized fact as the U.S.: long expansions tend to be followed by short contractions (Chart 9).[25] As in the U.S., there is a statistically significant negative relationship between the length of contractions and the length of preceding expansions. The results are similar to the postwar U.S. results: an additional month of expansion tended to be associated with roughly a one-sixth month reduction in the following contraction. However, the explanatory power of the regression using Canadian data (.30) is less than that using the postwar U.S. data (.38).

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[25] Two major business cycles were excluded from the U.S. analysis over the time period for which Canadian data are available. Since these two cycles for the U.S. and Canada correspond closely to one another, the two corresponding business cycles (1961-1969 and 1982-1989) are also excluded from the Canadian analysis.

Chart 8: Durations of Neighbouring Contractions and Expansions in U.S. postwar data

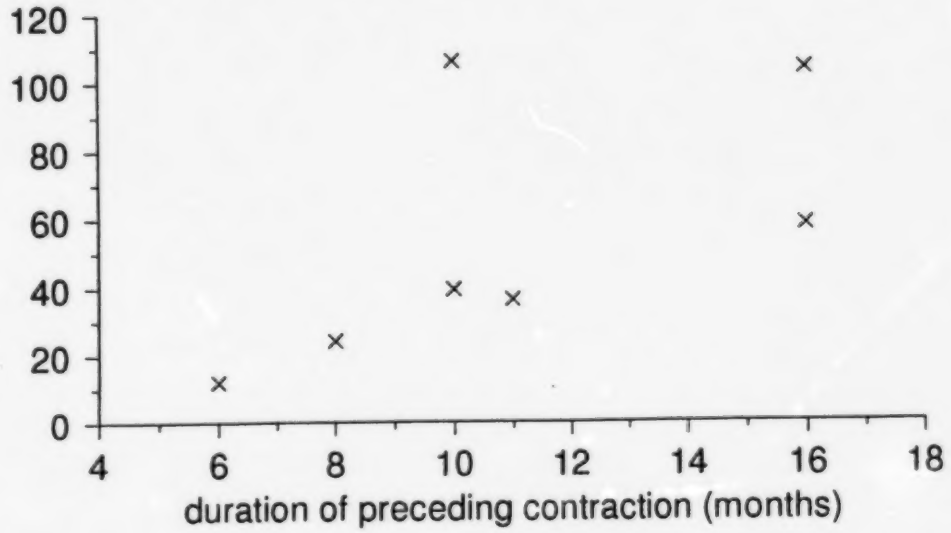


Chart 9: Durations of Neighbouring Expansions and Contractions in Canada

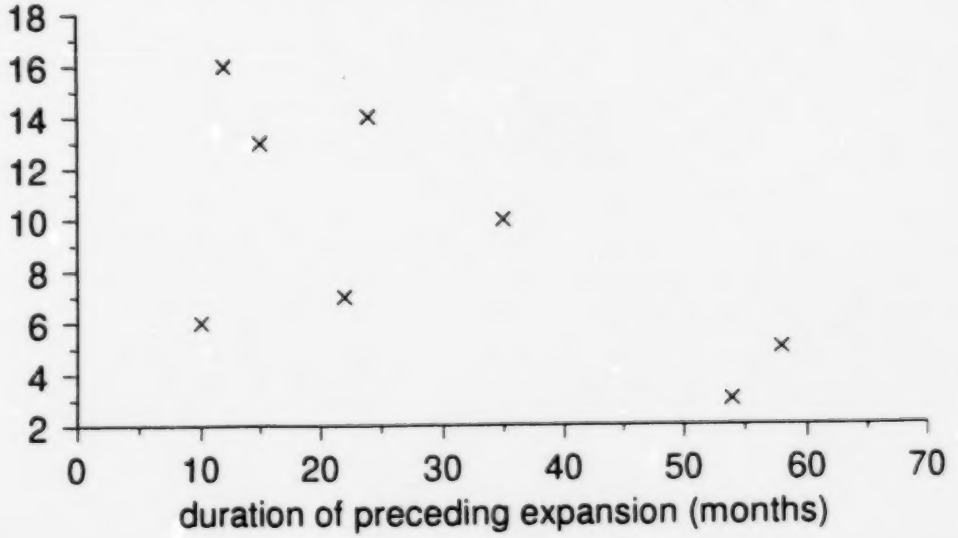


Table 20: Canadian Estimation Results, Dependent Variable: Contraction Length

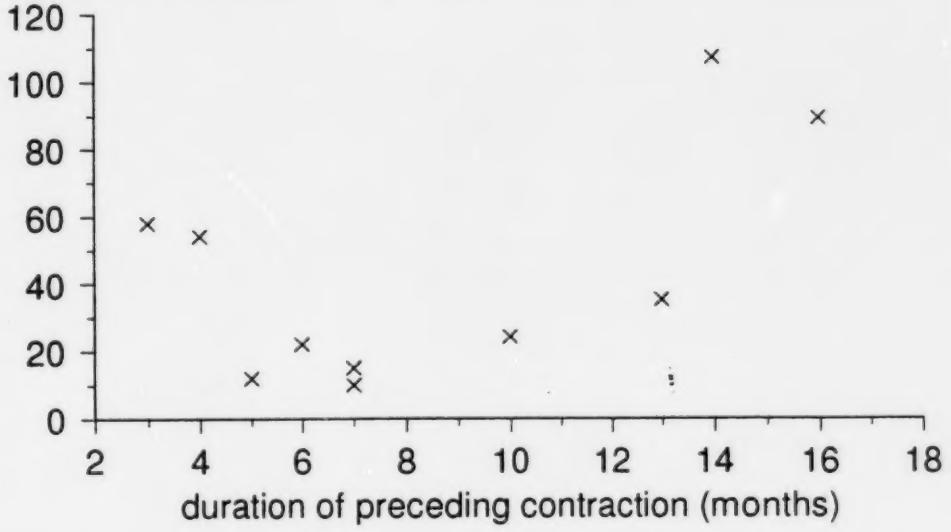
Intercept	13.9 (p=.00)
Expansion length	-.16 (p=.09)
Observations	8
$\overline{R}^2$	.30

Graphical analysis of the Canadian business cycle durations suggests there may also be a relationship between the length of an expansion and the length of the preceding contraction (Chart 10). The OLS regression results indicate a positive relationship between the length of a contraction and the length of the following expansion, although the explanatory power of the regression is not very high. As in the U.S., an additional month of contraction tends to be associated with a 5-month increase in the length of the following expansion (based on the results of the regression excluding an intercept term). Therefore, these results indicate that the length of the current expansion is not out of the ordinary, given the length of the most recent contraction.

Table 21: Canadian Estimation Results, Dependent Variable: Expansion Length

	Including intercept term	Excluding intercept term
Intercept	7.5 (p=.73)	
Contraction length	4.1 (p=.10)	4.8 (p=.00)
Observations	10	10
$\overline{R}^2$	.22	.29

Chart 10: Durations of Neighbouring Contractions and Expansions in Canada



V. Conclusion

The question of whether the length of an expansion increases the probability of a recession has become very relevant recently as the current expansion continues in both Canada and the U.S. This paper examined whether there is any credibility to the notion that the expansive phase of the business cycle is more likely to end as time passes.

Three different issues were analysed in this paper. The first is duration dependence, which specifically questions whether or not the probability of terminating one state of a business cycle varies with the duration to date of the state. *The majority of research has found that U.S. expansions after World War II do not exhibit statistically significant duration dependence.* In other words, contrary to popular opinion, the length of an expansion in the U.S. does not significantly increase the probability of a recession. However, prewar U.S. expansions and postwar U.S. contractions do tend to exhibit statistically significant positive duration dependence.

Some preliminary empirical work completed in this paper using Canadian business cycles indicated that, although there are signs of positive duration dependence, neither expansions nor contractions in Canada exhibit statistically or economically significant duration dependence. These results suggest that, in both Canada and the U.S., the risk of a recession is not significantly increasing with the length of the current recovery.

The second issue examined was whether the pattern of business cycle durations has changed over time. In the U.S., it is generally argued (using parametric and non-parametric techniques) that expansions have become significantly longer in the postwar period and contractions significantly shorter. However, there is not only evidence that the average durations have changed since the war, but that the durations have exhibited a trend shift over time. In other words, U.S. expansions have gradually lengthened and contractions have gradually shortened.

While the Canadian data do not allow for a prewar and postwar comparison, analysis was completed on two sub-samples of Canadian and U.S. durations, those between 1947 and 1960 and those between 1961 and 1990. In both Canada and the U.S., the average duration of expansions has increased sharply from the first sample to the second, but the average duration of contractions has remained fairly steady. As in the U.S., expansions in Canada have gradually lengthened over time. These results for Canada and the U.S. suggest that comparing current or recent durations of expansionary phases to earlier ones is indeed misleading.

The third issue examined is whether the length of one phase of a business cycle affects the length of the next. The estimation results of this paper indicate that, in both the U.S. and Canada, long contractions tend to be followed by long expansions. Given that the most recent recession in Canada (from April 1990 to April 1992) marked the longest recession recorded since business cycles have been dated, this provides further evidence that the length of the current expansion is not out of the ordinary.

To summarise, the evidence presented in this paper suggests that the probability of a recession in Canada or in the U.S. is not increasing significantly as the current expansion continues. Caution must also be taken in drawing inferences about current and recent business cycle durations based on the average of previous durations. These conclusions reflect the absence of duration dependence in Canadian expansions and U.S. postwar expansions, evidence that expansions are lengthening over time, and evidence that long contractions tend to be followed by long expansions. As Zarnowitz (1992) concluded, after his analysis of whether expansions die of old age, "What matters primarily, then, is not the passage of calendar time but what happens over time in and to the economy in motion. It is historical and psychological time, filled with events and processes, perceptions and actions."

Annex

Table 1: NBER U.S. Business Cycle Chronology[26]

Cycle Number	Expansion Begins	Length (months)	Contraction Begins	Length (months)
PREWAR				
1	Dec. 1854	30	June 1857	18
2*	Dec. 1858	22	Oct. 1860	8
3*	June 1861	46	Apr. 1865	32
4	Dec. 1867	18	June 1869	18
5*	Dec. 1870	34	Oct. 1873	65
6*	Mar. 1879	36	Mar. 1882	38
7	May 1885	22	Mar. 1887	13
8	Apr. 1888	27	July 1890	10
9	May 1891	20	Jan. 1893	17
10	June 1894	18	Dec. 1895	18
11	June 1897	24	June 1899	18
12	Dec. 1900	21	Sep. 1902	23
13	Aug. 1904	33	May 1907	13
14	June 1908	19	Jan. 1910	24
15	Jan. 1912	12	Jan. 1913	23
16	Dec. 1914	44	Aug. 1918	7
17*	Mar. 1919	10	Jan. 1920	18
18	July 1921	22	May 1923	14
19	July 1924	27	Oct. 1926	13

[26] The cycles marked with an asterisk (*) are classified as major cycles.

Cycle Number	Expansion Begins	Length (months)	Contraction Begins	Length (months)
20*	Nov. 1927	21	Aug. 1929	43
21*	Mar. 1933	50	May 1937	13
22*	June 1938	80	Feb. 1945	8
POSTWAR				
23	Oct. 1945	37	Nov. 1948	11
24	Oct. 1949	45	July 1953	10
25	May 1954	39	Aug. 1957	8
26	Apr. 1958	24	Apr. 1960	10
27*	Feb. 1961	106	Dec. 1969	11
28	Nov. 1970	36	Nov. 1973	16
29	Mar. 1975	58	Jan. 1980	6
30	July 1980	12	July 1981	16
31*	Nov. 1982	104	July 1990	8
32	Mar. 1991			

Annex

Table 2: Statistics Canada Business Cycle Chronology for Canada

Cycle Number	Expansion Begins	Length (months)	Contraction Begins	Length (months)
1	Apr. 1948	10	Feb. 1949	6
2	Aug. 1949	22	June 1951	7
3	Jan. 1952	15	Apr. 1953	13
4	May 1954	35	Apr. 1957	10
5	Feb. 1958	24	Feb. 1960	14
6	Apr. 1961	107	March 1970	4
7	July 1970	54	Jan. 1975	3
8	Apr. 1975	58	Feb. 1980	5
9	July 1980	12	July 1981	16
10	Nov. 1982	89	Apr. 1990	25
11	May 1992			

Annex

Table 3: Statistics Canada Original Business Cycle Chronology for Canada

Cycle Number	Contraction Begins	Length (months)	Expansion Begins	Length (months)
1	June 1951	7	Jan. 1952	17
2	June 1953	13	July 1954	31
3	Feb. 1957	12	Feb. 1958	26
4	Apr. 1960	10	Feb. 1961	160
5	June 1974	10	Apr. 1975	55
6	Nov. 1979	8	July 1980	12

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